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General estimation equation of transient $C(t)$ under load and displacement control

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Abstract

This paper propose estimation equations of transient $C(t)$-integrals for general material properties where plastic and creep stress exponent are different under load and displacement control. The new equations are made by modifying the plasticity correction term in the existing equations. The modified plasticity corrections term is expressed in terms of initial elastic-plastic and steady state creep stress fields. For validation, elastic-plastic-creep finite element analysis are performed. FE results are compared with predicted $C(t)$ results using proposed equations. Good agreement with FE results is found even when plastic and creep stress exponents are different.

Keywords: Transient $C(t)$-integrals; Elastic-plastic-creep; Crack-tip stress fields; Load and displacement control

1. Introduction

Creep crack growth is important factor into life assessment of components operating at high temperature. Creep crack growth rate can be quantified by the $C(t)$-integral which characterizes the singular stress and strain fields at the crack tip (Riedel, 1987). Note that the notation C^* is used for the value of $\underline{C}(t)$ at the steady state creep conditions.

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Nomenclature

a	crack length
A, B	material constants (plasticity and creep)
$C(t)$, C^*	C-integrals (transient and steady state creep)
D	normalized opening stress at t=0 (initial conditions), HRR fields
E	Young's modulus
F	normalized opening stress at $t\rightarrow\infty$ (steady state creep conditions), RR fields
$J(0)$	J-integrals for initial (t=0) conditions
L_r	parameter related to plastic yielding
m	strain hardening exponent
n	creep exponent
M, M_L	applied load and plastic limit load
r, θ	polar coordinates at the crack tip
t, t_{red}	time and redistribution time
x, y	Cartesian coordinates
Z	elastic follow-up factor
ε, ε^e, ε^p	strain, elastic strain and plastic strain
v	Poisson's ratio
τ	normalized time, $=t/t_{red}$
σ	stress
σ_o	yield strength
σ_{ref}	reference stress
φ	plasticity correction factor under load control
γ	plasticity correction factor under displacement control
Φ	parameter related to elastic follow-up, $=Z/(Z-1)$

Thus estimations of $C(t)$ and C^* are needed to assess creep crack growth in conjunction with creep crack growth rate data determined in terms of $C(t)$ and C^* from specimen tests. For elastic-power law creep problem, Ehlers and Riedel (1981) proposed a $C(t)/C^*$ relaxation curve. A slightly different equation was developed by Ainsworth and Budden (1990). However, the approach can invalidate under widespread plasticity. For widespread plasticity, Joch and Ainsworth (1992) presented the effect of initial plasticity on the magnitude of $C(t)$-integral during the transient creep. Based on the approach of Ainsworth and co-workers, Lei (2005) proposed equation of $C(t)/C^*$ relaxation curve for secondary loading cases. Note that above equations (Joch and Ainsworth, 1992; Lei, 2005) which take account of initial plasticity are valid only for equal power law stress exponents, i.e., the plastic hardening exponent (m) and creep exponent (n) are the same. Generally, materials have unequal stress exponents for plasticity and creep. Therefore, a more general equation is needed to apply for general stress exponent cases. The present work presents estimation equation of transient $C(t)$ for general elastic-plastic-creep conditions where the plastic and creep exponents are different under load and displacement control. The new equation is made by modifying the plasticity-correction term in the existing equations. The proposed equations are validated against elastic-plastic-creep finite element (FE) analysis results for plane strain single-edge-cracked bend (SE(B)) specimen.

2. Finite element analysis

2.1. Geometry

One typical geometry with high crack-tip constraint levels was considered in this paper: plane strain single-edge-cracked bend (SE(B)) specimen, as depicted in Fig. 1. The specimen width, W, was taken to be W=50mm with the relative crack depth a/W=0.5

In Fig.1, r and θ denote polar coordinated at the crack tip; y denotes crack opening direction.

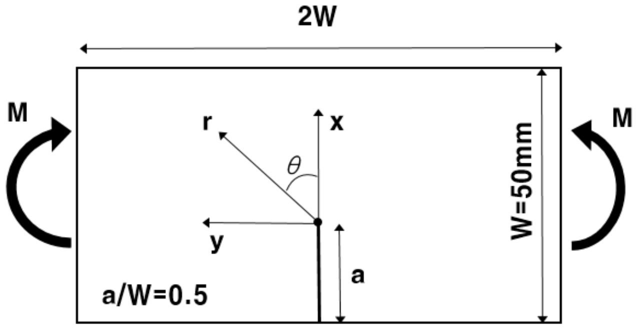

Fig. 1. Specimen conceded in this paper, schematics: SE(B).

2.2. Material properties

For elastic-plastic analyses, an isotropic material was assumed to follow the Ramberg-Osgood relationship:

$$\varepsilon = \varepsilon^e + \varepsilon^p = \frac{\sigma}{E} + \alpha\varepsilon_o\left(\frac{\sigma}{\sigma_o}\right)^m \quad \text{with } \alpha = \frac{0.002E}{\sigma_o} \quad \text{and } \varepsilon_o = \frac{\sigma_o}{E}$$
$$= \frac{\sigma}{E} + A\sigma^m \tag{1}$$

where ε, ε^e, ε^p denote total, elastic, plastic strain, respectively; σ is stress (MPa); A and m are material constants. For elastic properties, Young's modulus E=200GPa and Poisson's ratio ν=0.3 were used. For plastic properties, the yield strength σ_o was assumed to be 300MPa with two values of the strain hardening exponent, m=5 and 10.

For creep analyses, the material was assumed to follow power-law behavior, characterized by:

$$\dot{\varepsilon}^c = B\sigma^n \tag{2}$$

where $\dot{\varepsilon}^c$ denote creep strain rate; B and n are material constants. Two values of the creep exponents n were considered, n=5 and 10. The following values were assumed, B=3.2x10^{-15} (MPa)$^{-n}h^{-1}$ for n=5 and B=3.2x10^{-25} for n=10. However, the values of constant B don not affect the results as these are presented in a normalized manner.

2.3. Finite element analysis

Elastic-plastic-creep Fe analyses of SE(B) specimen were performed using ABAQUS (2013).To avoid problems associated with incompressibility, eight-noded plane strain element with reduced integration were used. A small geometry change continuum FE model was assumed. Figure 2 depicts the FE mesh for SE(B) specimen. The crack-tip was designed with collapsed elements, and a ring of wedge-shaped elements was used in the crack-tip region. The number of elements and nodes in the FE meshes were 4543 and 14055.

To apply pure bending loading conditions, the multi-point constraint (MPC) option within ABAQUS was used. To quantify the applied loading magnitude, a parameter related to plastic yielding, L_r, is used:

$$L_r = \frac{M}{M_L} = \frac{\sigma_{ref}}{\sigma_o} \tag{3}$$

$$M_L = \frac{1.261}{2\sqrt{3}} b\left(W-a\right)^2 \sigma_o \quad \text{for SE(B)} \tag{4}$$

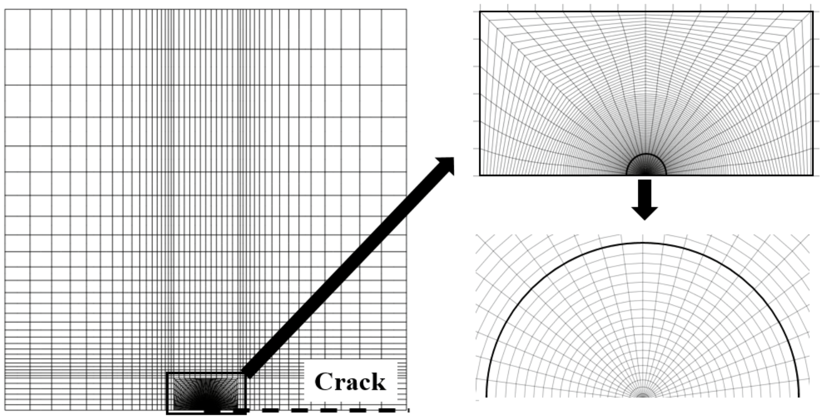

Fig. 2. FE mesh for SE(B) specimen.

where M and M_L denote the applied load and the plastic limit load based on the von Mises yield condition (Webster and Ainsworth, 1987); b is the specimen thickness; and σ_{ref} is the reference stress. In this work, three different values of L_r, L_r =0.5, 0.8 and 1.0, were considered.

Elastic-plastic-creep FE analyses were performed as follows. For load controlled cases, constant loading was applied in the first step (at time t=0). The load was then held constant for t>0. For displacement controlled cases, constant displacement which related to L_r within load control was applied in the first step (at time t=0). The displacement was then held constant for t>0 and subsequent time-dependent creep calculations were performed. For time-dependent creep calculations, an implicit method was selected within ABAQUS.

3. Transient *C*(*t*) estimation

3.1. Existing transient C(t) estimation equation

For elastic-creep conditions under load control, Ehlers and Riedel (1981) proposed a relaxation curve for $C(t)$ in terms of time t and steady-state C^*:

$$\frac{C(t)}{C^*} = 1 + \frac{1}{(n+1)} \frac{t}{t_{red}} = 1 + \frac{\tau}{(n+1)} \tag{5}$$

where τ denotes the normalized time which is given in terms of redistribution time t_{red}

$$\tau = \frac{t}{t_{red}} = \frac{C^* t}{J(0)} \tag{6}$$

$J(0)$ denotes the initial (at time t=0) FE value of J-integrals.

For elastic-plastic-creep conditions under load control, Joch and Ainsworth (1992) proposed another equation that the effect of initial plasticity on $C(t)$ could be incorporated using a factor φ:

$$\frac{C(t)}{C^*} = \frac{(1+\tau)^{n+1}}{(1+\tau)^{n+1} - \phi} \quad \text{with } \phi = 1 - \frac{AC^*}{BJ(0)} \tag{7}$$

The material constant for plasticity and creep, A and B, are given in Eqs. (1) and (2), respectively.

For elastic-plastic-creep conditions under displacement control, Lei (2005) proposed equation of $C(t)/C^*$ relaxation curve based on the approach of Ainsworth and co-workers:

$$\frac{C(t)}{C^*}=\frac{\left(\frac{\sigma_{ref}}{\sigma_{ref}^o}\right)^{n+1}\left(\frac{\varepsilon_{ref}}{\varepsilon_{ref}^o}\right)^{n+1}}{\Phi\left[\left(\frac{\varepsilon_{ref}}{\varepsilon_{ref}^o}\right)^{n+1}-1\right]+\gamma}\quad \text{With}\ \left(\Phi=\frac{Z}{Z-1}\right)\ \text{and}\ \left(\gamma=1-\frac{\sigma_{ref}^o}{E\varepsilon_{ref}^o}\right) \tag{8}$$

where Z denotes elastic follow-up factor. The following values were assumed, Z=2.0 for n=5 and Z=2.5 for n=10. An important point to note is that Eqs. (7) and (8) were derived based on the assumption of equal stress exponents for plasticity and creep (m=n). When the stress exponent are different ($m\neq n$), Eqs. (7) and (8) cannot be applied.

3.2. Proposed transient C(t) estimation equation

A new estimation equation is made by changing plasticity correction term φ, γ in terms of the crack-tip stress fields at the initial and steady state creep conditions. At initial conditions (time t=0), the crack-tip stress field should follow the Hutchinson-Rice-Rosengren field (1968), and is denoted as D:

$$\left(\frac{\sigma_{yy}}{\sigma_o}\right)_{t=0}=\left[\frac{J(0)}{I_m A\sigma_o^{m+1}r}\right]^{\frac{1}{m+1}}\tilde{\sigma}_{yy}(m,\theta)\equiv D \tag{9}$$

where r and θ denote polar coordinate at the crack-tip. At t>0, the crack-tip stress under creep conditions is given by:

$$\frac{\sigma_{yy}}{\sigma_o}=\left[\frac{C(t)}{I_n B\sigma_o^{n+1}r}\right]^{\frac{1}{n+1}}\tilde{\sigma}_{yy}(n,\theta) \tag{10}$$

where I_m (or I_n) is an constant that depend on stress exponent. At long times under steady-state creep conditions, the crack-tip stress field follow the RR field (Riedel and Rice, 1980), and is denotes as F:

$$\left(\frac{\sigma_{yy}}{\sigma_o}\right)_{t\to\infty}=\left[\frac{C^*}{I_n B\sigma_o^{n+1}r}\right]^{\frac{1}{n+1}}\tilde{\sigma}_{yy}(n,\theta)\equiv F \tag{11}$$

For load controlled cases, using Eqs. (7) and (11), Eq. (10) can be re-written as

$$\frac{\sigma_{yy}}{\sigma_o}=\frac{F(1+\tau)}{\left[(1+\tau)^{n+1}-\phi\right]^{\frac{1}{n+1}}} \tag{12}$$

Equation (12) is crack-tip stress field at transient creep condition under load control. By matching Eq. (9) and Eq. (12) at time t=0, we can obtain that

$$\frac{C(t)}{C^*}=\frac{(1+\tau)^{n+1}}{(1+\tau)^{n+1}-\phi'}\quad \text{with}\ \phi'=1-\left(\frac{F}{D}\right)^{n+1}\quad (\text{If}\ \phi'<0,\ \text{then}\ \phi'=0) \tag{13}$$

Equation (13) is the proposed $C(t)$ estimation equation under load control. Eq. (13) has the same form as Eq. (7), but the plasticity-correction factor is different. In the cases of m=n, the equations are the same.

For displacement controlled cases, using Eqs. (8) and (11), Eq. (10) can be re-written as

$$\frac{\sigma_{yy}}{\sigma_o} = \frac{F\left(\frac{\sigma_{ref}}{\sigma_{ref}^o} \frac{\varepsilon_{ref}}{\varepsilon_{ref}^o}\right)}{\left\{\Phi\left[\left(\frac{\varepsilon_{ref}}{\varepsilon_{ref}^o}\right)^{n+1} - 1\right] + \gamma\right\}^{\frac{1}{n+1}}} \tag{14}$$

Equation (14) is crack-tip stress field at transient creep condition under displacement control. By matching Eq. (9) and Eq. (14) at time t=0, we can obtain that

$$\frac{C(t)}{C^*} = \frac{\left(\frac{\sigma_{ref}}{\sigma_{ref}^o}\right)^{n+1}\left(\frac{\varepsilon_{ref}}{\varepsilon_{ref}^o}\right)^{n+1}}{\Phi\left[\left(\frac{\varepsilon_{ref}}{\varepsilon_{ref}^o}\right)^{n+1} - 1\right] + \gamma'} \quad \text{with } \gamma' = \left(\frac{F}{D}\right)^{n+1} \tag{15}$$

Equation (15) is the proposed $C(t)$ estimation equation under displacement control. Eq. (15) has the same form as Eq. (8), but the plasticity-correction factor is different. In the cases of m=n, the equations are the same.

3.3. Validation

Elastic-plastic values of J-integral at t=0, $J(0)$, and elastic-plastic-creep values of C-integral at steady state creep, C^*, are determined from FE analysis. Determined values of $J(0)$ and C^* are presented in Table 1. Using determined $J(0)$ and C^*, values of factor φ' in Eq. (13) and factor γ' in Eq. (15) are calculated. The proposed $C(t)$ estimation equations are compared with the FE results in Fig. 3 (for load control) and Fig. 4 (for displacement control). Although the prediction is slightly non-conservative for the case of m=10, n=5 with L_r=1.0 in Fig. 3, overall $C(t)/C^*$ relaxation curves using the new equation agree well with FE results.

Table 1. Values of $J(0)$ and C^* from FE analysis.

	$J(0)$ (MPa·mm)			C^* (MPa·mm/h)		
	L_r			L_r		
	0.5	0.8	1.0	0.5	0.8	1.0
m=n=5	6.03	20.20	42.40	1.06	17.85	68.07
m=n=10	5.67	17.92	42.39	6.84	1203	14001
m=5, n=10	6.03	20.20	42.40	6.84	1203	14001
m=10, n=5	5.67	17.92	42.39	1.06	17.85	68.07

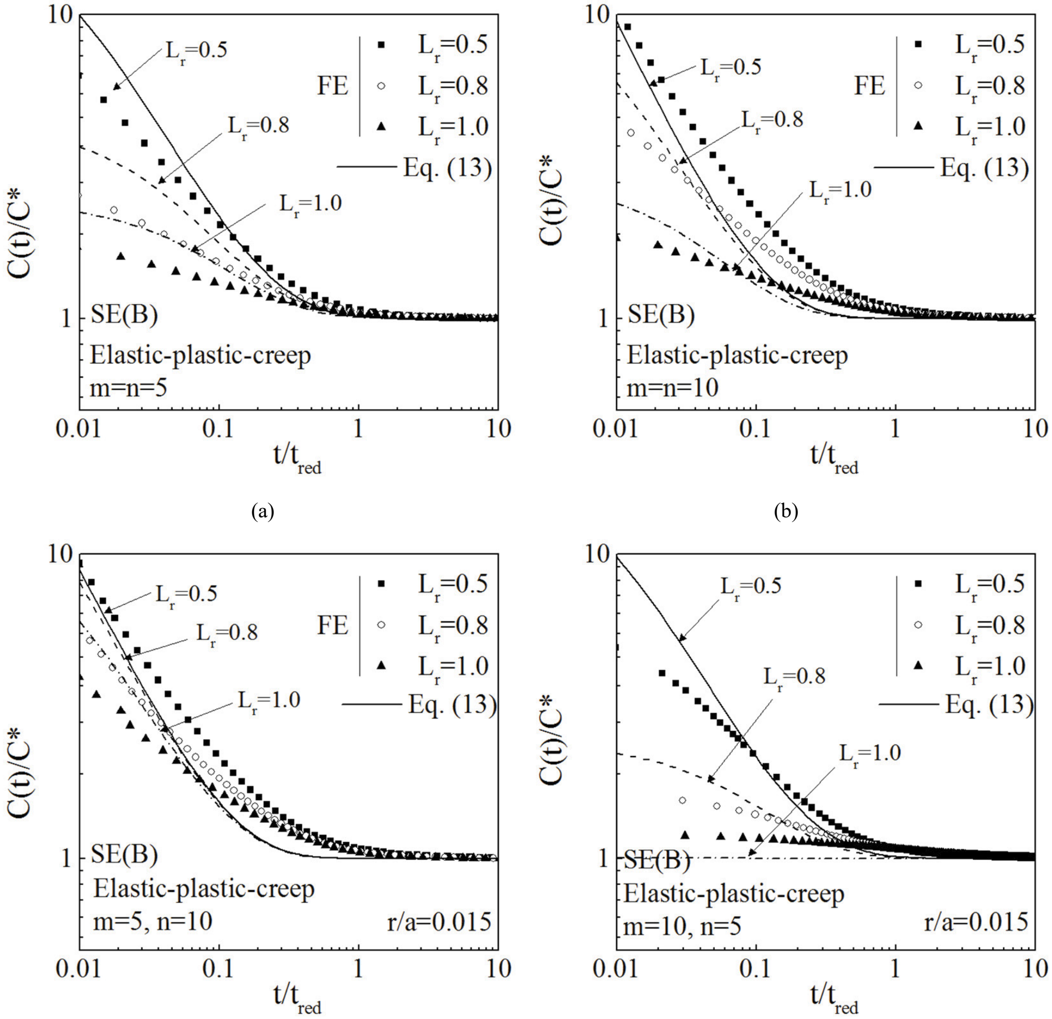

Fig. 3. Variations of $C(t)/C^*$ for load controlled cases: (a) m=n=5, (b) m=n=10, (c) m=5, n=10, and (d) m=10, n=5.

4. Conclusions

In this work, estimation equation for transient $C(t)$ are proposed for general material where the plastic and creep stress exponents are different under load and displacement control. The new equations are expressed in terms of crack-tip stress fields at initial elastic-plastic and steady-stated creep conditions. These can be calculated from analytical HRR and RR field expression. To validate the proposed equations, the predicted $C(t)$ values are compared with elastic-plastic-creep FE results for plane strain single-edge-crack bend specimen. It is found that the proposed equations provide good agreement with the FE results, even when plastic and creep stress exponent are different. The present results can provide insight on the estimation of transient $C(t)$ under load and displacement control.

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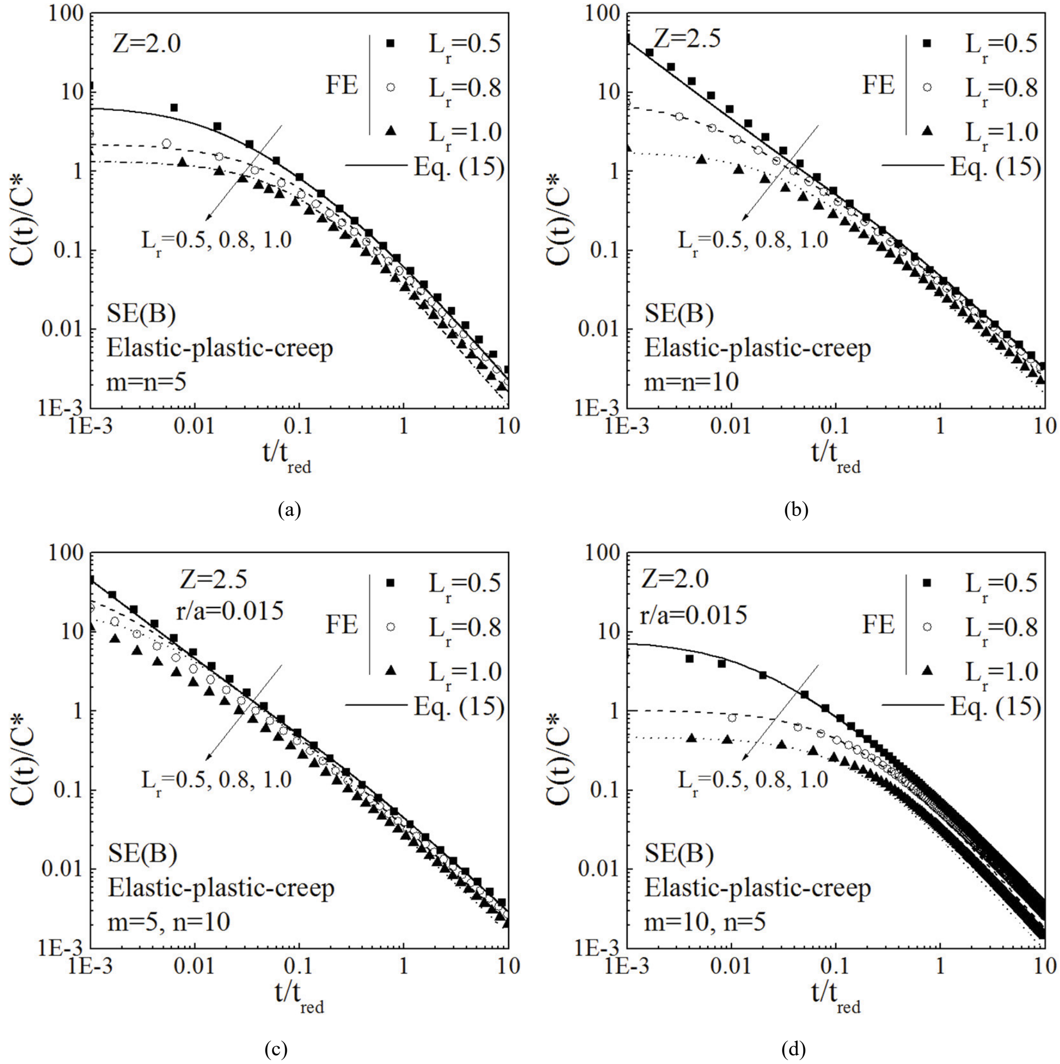

Fig. 4. Variations of $C(t)/C^*$ for displacement controlled cases: (a) m=n=5, (b) m=n=10, (c) m=5, n=10, and (d) m=10, n=5.

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